B.Sc. (CBCS Pattern) Semester-IV

USMT-08 - Mathematics Paper-II - Elementary Number Theory

P. Pages : 2 **GUG/S/25/12015**

Time : Three Hours Max. Marks : 60

* 1 8 2 1 *

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT-I

1. a) Find all positive integers $n\,(<17)$ for which $n!+(n+1)!+(n+2)!$ is an integral multiple of 49. **6**

b) Prove that, the product of any m consecutive integers is divisible by m! **6**

OR

c) Find the values of x and y to satisfy the equation $423x+198y=9$. **6**

d) For a positive integers a and b prove that $(a,b)[a,b]=ab$. **6**

UNIT-II

2. a) Prove that every positive integer greater than one has atleast one prime divisors. **6**

b) If $(a,b)=1$, Then show that $(a^2,b^2)=1$. **6**

OR

c) Prove that, for any positive integer n there are at least n consecutive composite integers. **6**

d) Prove that, any two distinct Fermat number are relatively prime. **6**

UNIT-III

3. a) Let a, b, c be integers such that $a\equiv b\,(\text{mod}\, m)$ then prove that **6**

i) $(a-c)\equiv(b-c)\,(\text{mod}\, m)$

ii) $ac\equiv bc\,(\text{mod}\, m)$

b) Prove that, congruence is an equivalence relation. **6**

OR

c) Solve the congruence equation **6**
$7x\equiv 3\,(\text{mod}\,12)$

d) Solve the system of three congruences **6**
$x\equiv 1\,(\text{mod}\,4), x\equiv 0\,(\text{mod}\,3), x\equiv 5\,(\text{mod}\,7)$

UNIT-IV

4. a) If P is a prime integer with $\phi(p) = p-1$ then prove that P is prime. **6**

b) Show that the sum of $\phi(n)$ positive integers less than $n\,(>1)$ and relatively prime to n is $\frac{n}{2}\phi(n)$. **6**

OR

c) Prove that, for $n > 2, \phi(n)$ is an even integer. **6**

d) Let m be a positive integer and "a" be an integer with $(a, m) = 1$ then prove that $a^{\phi(m)} \equiv 1 (\operatorname{mod} m)$. **6**

5. Solve **any six.**

a) Let a and b be any two integers that are not both zero then prove that their g.c.d. is unique. **2**

b) Prove that, if c/a and c/b then $c/(a, b)$. **2**

c) Define prime number and composite number. **2**

d) Prove that, if $2^m - 1$ is prime then m is also a prime. **2**

e) Let a and b be integers then prove that $a \equiv b\,(\operatorname{mod} m)$ if and only if there is a integer k such that $a = b + k.m$. **2**

f) Let $a_1, a_2, c \in Z$ then prove that $c\,a_1 \equiv c\,a_2 (\operatorname{mod} m)$. **2**

g) If $f(n) = 1$ and $g(n) = n, \forall n \in N$ then prove that f and g are completely multiplicative function. **2**

h) Find the value of $\phi(300)$. **2**

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